Momentum-Aware Trajectory Optimization and Control for Agile Legged Locomotion

Ziyi Zhou*, Bruce Wingo*, Nathan Boyd, Seth Hutchinson, and Ye Zhao

(*Equally contributed), Institute for Robotics and Intelligent Machines, Georgia Institute of Technology

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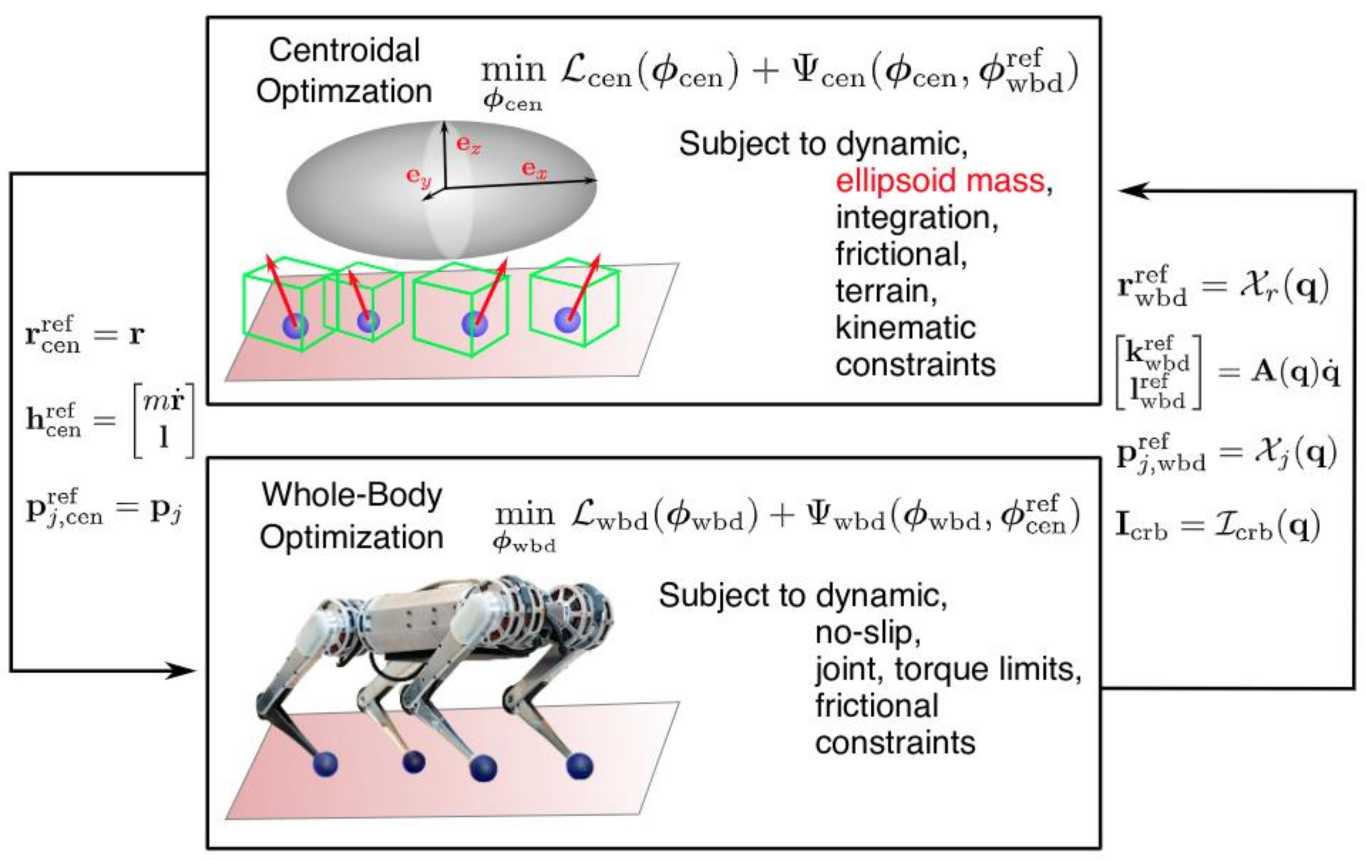
Introduction and Objective

- We develop a versatile hierarchical offline planning algorithm, along with an online control pipeline for agile quadrupedal locomotion.
- Propose **an optimization scheme that alternates between centroidal and whole-body optimization**, to generate a wide range of motions, including inertia shaping behaviors.
- Formulate **a convex model predictive controller** through a novel linear transformation of full centroidal dynamics to efficiently track momentum-rich motions.

Alternating CEN-WBD Optimization

- The dynamic consensus [1] is enforced by adding equality **consistency constraints for Center of Mass (CoM) positions, centroidal momentum, and footholds**. The Moment of Inertia (MoI) is directly computed from whole-body composite rigid body MoI.
- Centroidal optimization [2] utilizes an **equimomental-ellipsoid parameterization** to capture the change of MoI [3].
- Whole-body optimization solves a **constrained differential dynamic programming** (DDP) problem [4] using full order dynamics.



Model Predictive Control with Centroidal Dynamics

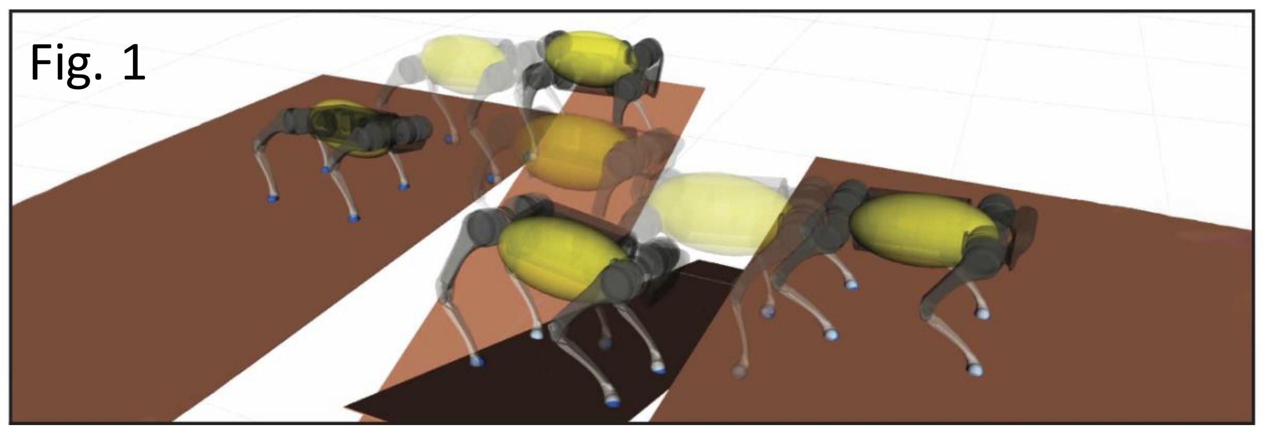
- An online **full centroidal convex MPC** re-computes foot contact forces and joint accelerations based on state feedback from the robot, and allows an explicit momentum tracking compared with original convex MPC [5].
- Based on the assumption that the robot follows the desired reference trajectory, we can pre-compute the matrix H and G inside the dynamics equation, which simplifies to a **linear time-varying** system.

$$\begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\mathbf{q}}_b \\ \dot{\mathbf{q}}_j \\ \ddot{\mathbf{q}}_j \\ \dot{\mathbf{g}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & m\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \\ \widetilde{\mathbf{A}}_h(\mathbf{q}) & \mathbf{0} & \mathbf{0} & \widetilde{\mathbf{A}}_j(\mathbf{q}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathbf{H}(\mathbf{q})} \begin{bmatrix} \mathbf{h} \\ \mathbf{q}_b \\ \mathbf{q}_j \\ \dot{\mathbf{q}}_j \\ \mathbf{g} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{I} & \cdots & \mathbf{I} & \mathbf{0} \\ \widehat{(\mathbf{p}_0 - \mathbf{r})} & \cdots & \widehat{(\mathbf{p}_{n_f-1} - \mathbf{r})} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathbf{G}(\mathbf{p},\mathbf{r})} \underbrace{\begin{bmatrix} \mathbf{f}_0 \\ \vdots \\ \mathbf{f}_{n_f-1} \\ \mathbf{a} \end{bmatrix}}_{\mathbf{u}}$$

Results

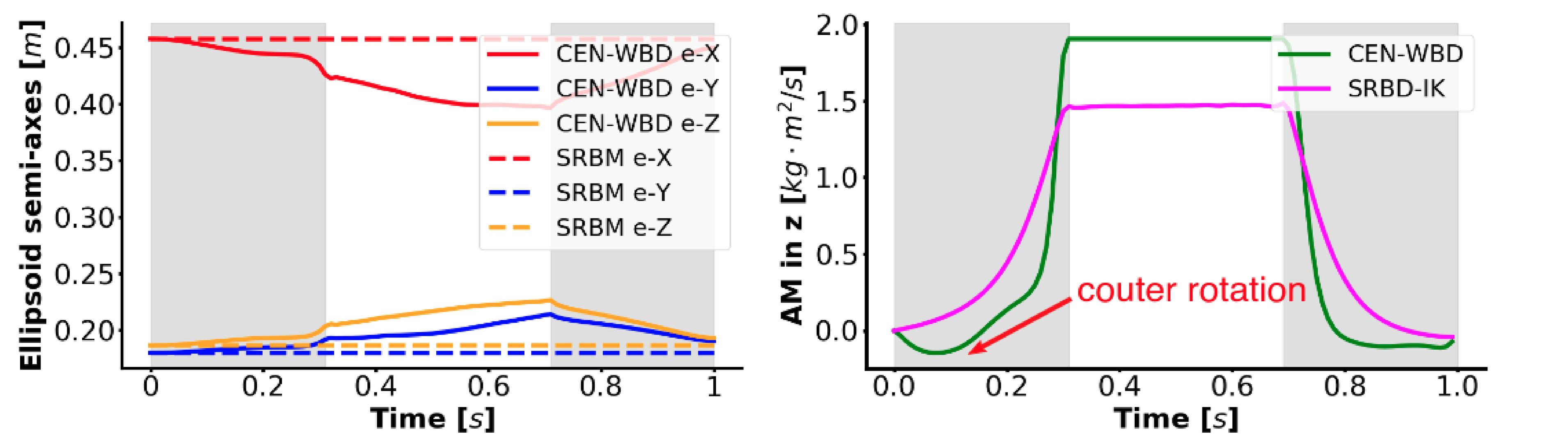
- Simulated and hardware tests conducted
- Simulated "parkour" maneuver, where the robot **leaps between platforms** (Fig. 1)

Fig. 1



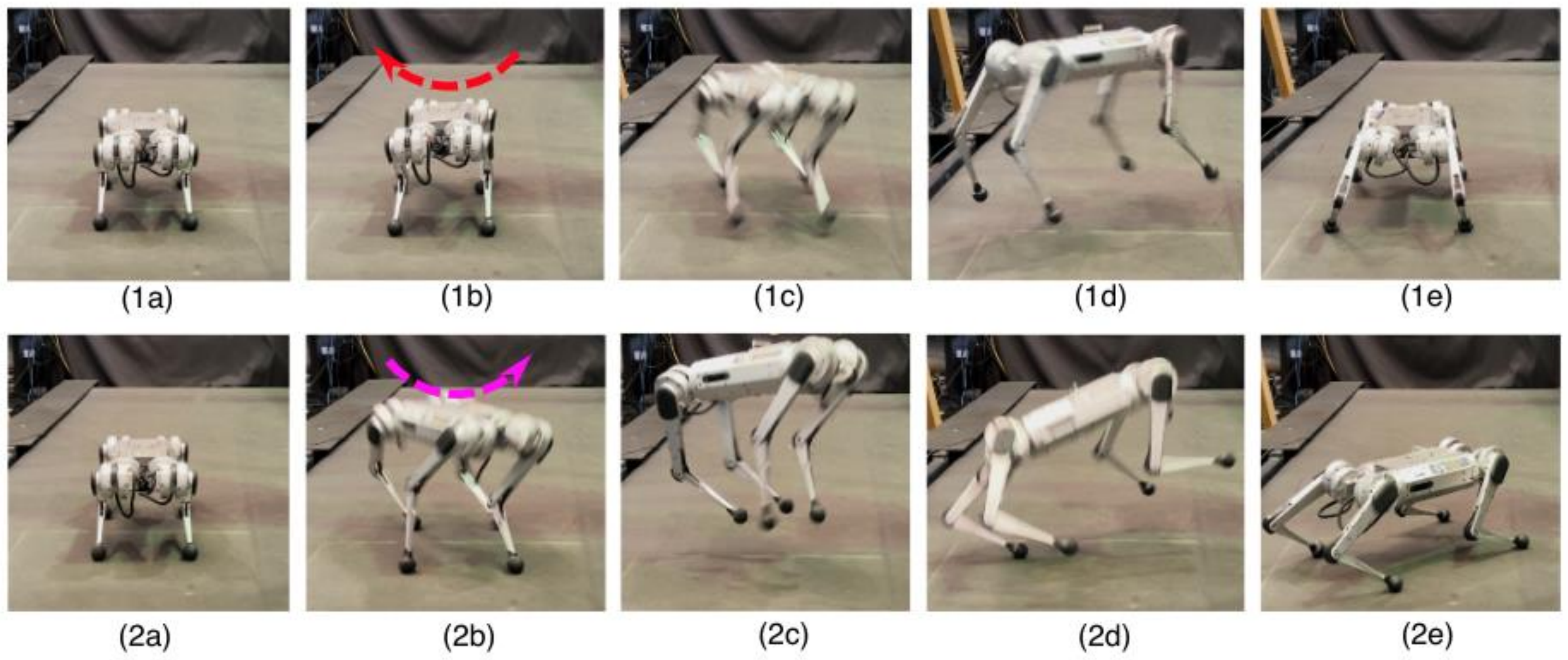
- Equimomental ellipsoid semi-axes changes (left) and centroidal AM in z direction (right) generated by CEN-WBD and SRBM+IK

Fig. 2



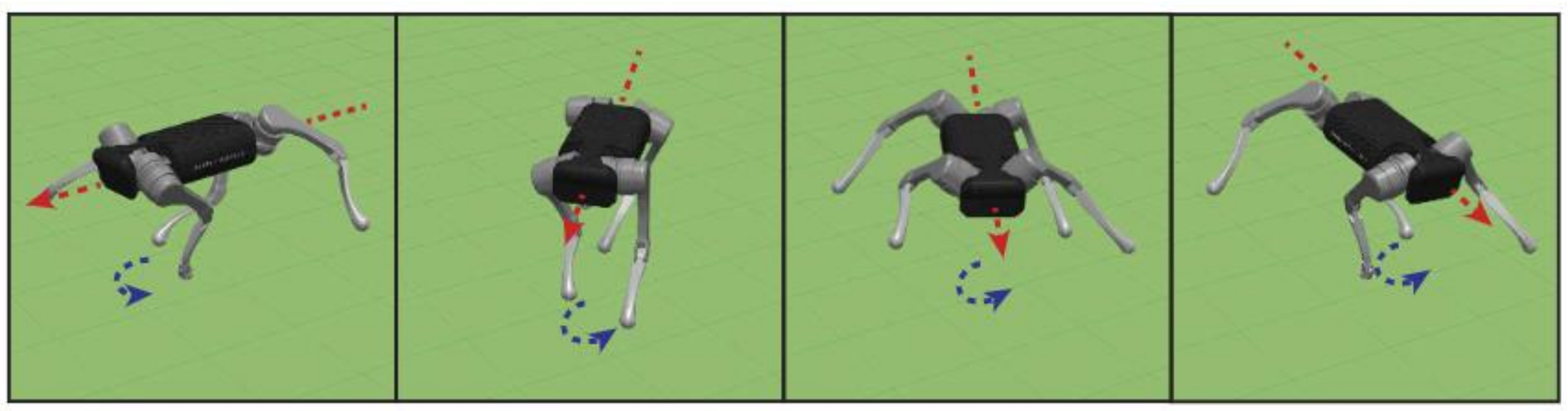
- Hardware demonstration of the MPC executing a jump-twist maneuver on hardware (Fig. 3); our CEN-WBD is shown in (1a - 1e) while the SRBM+IK is shown in (2a - 2e).

Fig. 3



- Simulation of a zero-gravity turn (Fig. 4) that is unsolvable without whole-body knowledge

Fig. 4



Future Work

- Future works include optimizing over foot placements and contact timings for both offline and online parts
- MPC controller that realizes cantering and "parkour" on hardware.

References

[1] H. Alexander et al, "Structured contact force optimization for kino-dynamic motion generation", 2016 **[2]** A. W. Winkler et al, "Gait and Trajectory Optimization for Legged Systems Through Phase-Based End-Effector Parameterization", 2018 **[3]** V. Zordan et al, "Control of Rotational Dynamics for Ground and Aerial Behavior", 2014 **[4]** C. Mastalli et al, "Crocoddyl: An Efficient and Versatile Framework for Multi-Contact Optimal Control", 2020 **[5]** J. Carlo, et al, "Dynamic Locomotion in the MIT Cheetah 3 Through Convex Model-Predictive Control", 2018