# Optimal Scheduling of Models and Horizons for Model Hierarchy Predictive Control

Charles Khazoom, Steve Heim, Daniel Gonzalez-Diaz and Sangbae Kim

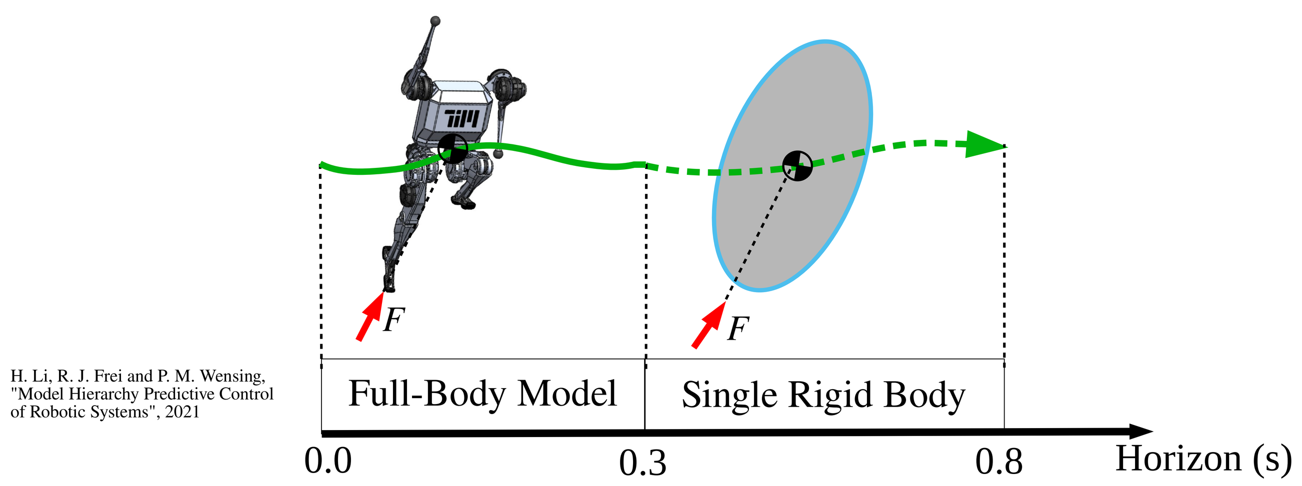
## Motivation

The computational demands of model predictive control (MPC) impose limitations on the dynamics model used for planning. Instead of using a single complex model along the MPC horizon, model hierarchy predictive control (MHPC) reduces solve times by planning over a *sequence* of models of *varying complexity* within a single horizon. Choosing this model sequence can become intractable when considering all possible combinations of reduced order models and prediction horizons.

**We propose a framework to systematically optimize the model sequence for MHPC.**

## Model Hierarchy Predictive Control

- Previously, MHPC has been implemented using a full-body model, followed by a single rigid body model:




- MHPC is formulated as a multiphase trajectory optimization (TO) with cost, dynamics and constraints varying at each stage $k$:

$$\hat{V}^{\boldsymbol{\mathcal{S}}}(\hat{\boldsymbol{x}}_1) = \min_{\hat{\boldsymbol{\mathcal{X}}}} \sum_{k=1}^{\hat{N}-1} \hat{c}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k) + \hat{c}_{\hat{N}}(\hat{\boldsymbol{x}}_{\hat{N}}) \quad (\mathcal{MHPC})$$

subject to

$$\left.\begin{aligned} &\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{f}}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k)\ \forall\ k \in \{1, ..., \hat{N}-1\} \\ &\hat{\boldsymbol{g}}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k) \leq \boldsymbol{0}\ \forall\ k \in \{1, ..., \hat{N}-1\} \end{aligned}\right\} \text{Time varying constraints to be optimized : The model sequence } \boldsymbol{\mathcal{S}}$$

$$\hat{\boldsymbol{g}}_N(\hat{\boldsymbol{x}}_N) \leq \boldsymbol{0},$$

## Problem Statement

The model sequence optimization is posed as a policy search problem over the model sequence of MHPC

$\boldsymbol{\mathcal{S}}$: model sequence

$\hat{\boldsymbol{\mathcal{X}}}$: Set of decision variables

$V^{\boldsymbol{\mathcal{S}}}$ : Closed-loop Cost of MHPC

$V^*$ : Optimal Cost

$$\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}}^* = \arg\min_{\boldsymbol{\mathcal{S}}} |\hat{\boldsymbol{\mathcal{X}}}| \qquad \textbf{Minimize the number of decision variables}$$

subject to

$$\frac{V^{\boldsymbol{\mathcal{S}}}(\boldsymbol{x}_1) - V^*(\boldsymbol{x}_1)}{V^*(\boldsymbol{x}_1)} \leq \epsilon \qquad \textbf{Closed-loop cost near optimal}$$

- We use a full-body trajectory optimization to estimate $V^*(\boldsymbol{x}_1)$
- We treat the second timestep of the MHPC predictions as a simulation step to estimate $V^{\boldsymbol{\mathcal{S}}}(\boldsymbol{x}_1)$

## From Full-Body to Reduced Order Dynamics

Instead of deriving the equations of motion for each reduced order model, we add holonomic constraints and constrain the full-body dynamics to behave like a reduced order model.

$$\boldsymbol{H}(\boldsymbol{q}_k)(\dot{\boldsymbol{q}}_{k+1} - \dot{\boldsymbol{q}}_k) + \boldsymbol{C}(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k)dt_k = \boldsymbol{B}\boldsymbol{u}_k dt_k$$

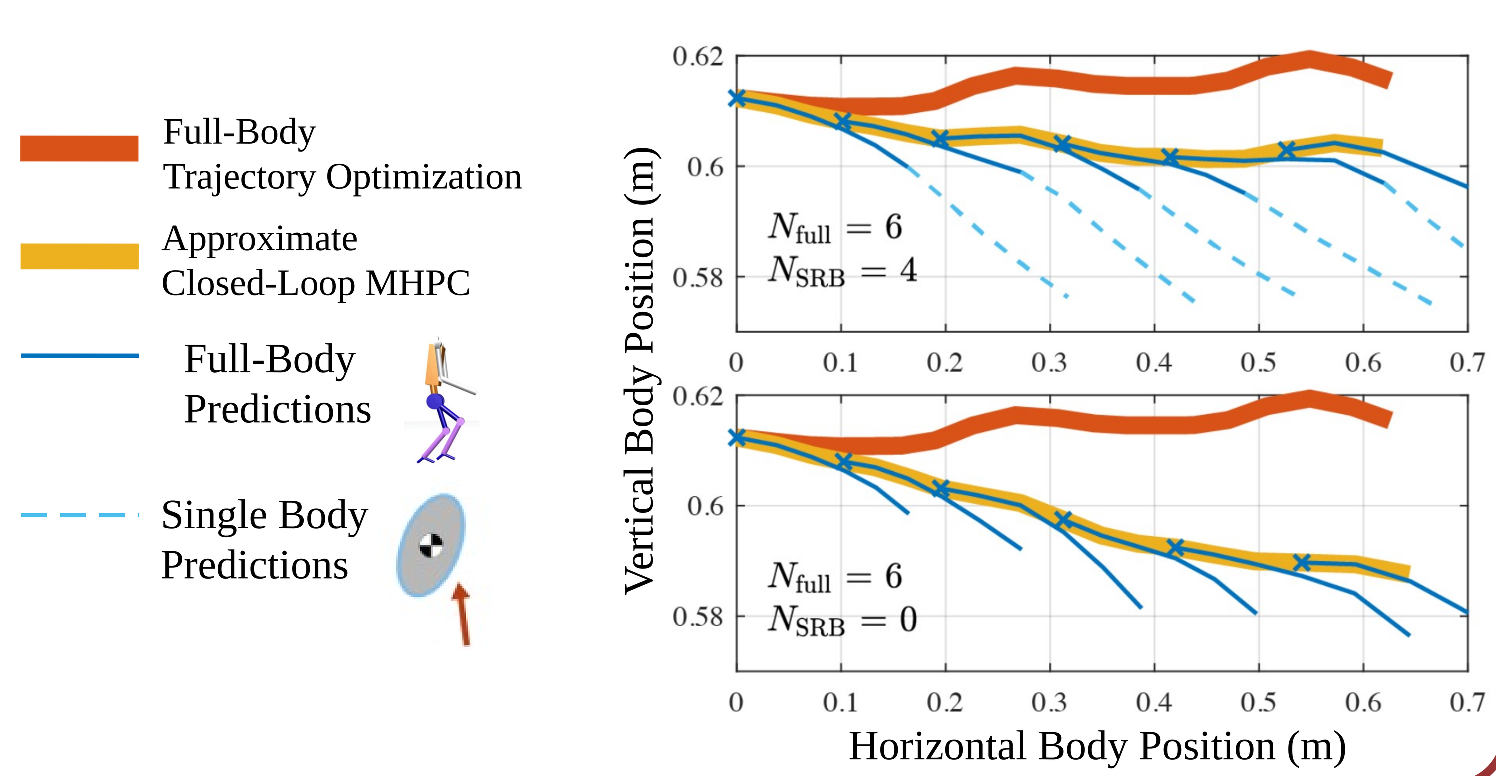
Foot contact constraints $\quad \big\{ + s_{\text{full},k}\boldsymbol{J}_c^\top(\boldsymbol{q}_k)\boldsymbol{F}_{c,k}$

The single rigid body model (SRB) locks all the joints. $\quad \left\{ \begin{aligned} &+ s_{\text{SRB},k}\boldsymbol{J}_{\text{SRB}}^\top(\boldsymbol{q}_k)\boldsymbol{F}_{\text{SRB},k} \\ &+ s_{\text{SRB},k}\boldsymbol{J}_{\text{fb}}^\top\boldsymbol{F}_c \end{aligned}\right.$

The void model truncates the horizon by locking joints and floating base $\quad \big\{ + s_{\text{void},k}\boldsymbol{J}_{\text{void}}^\top(\boldsymbol{q}_k)\boldsymbol{F}_{\text{void},k}$

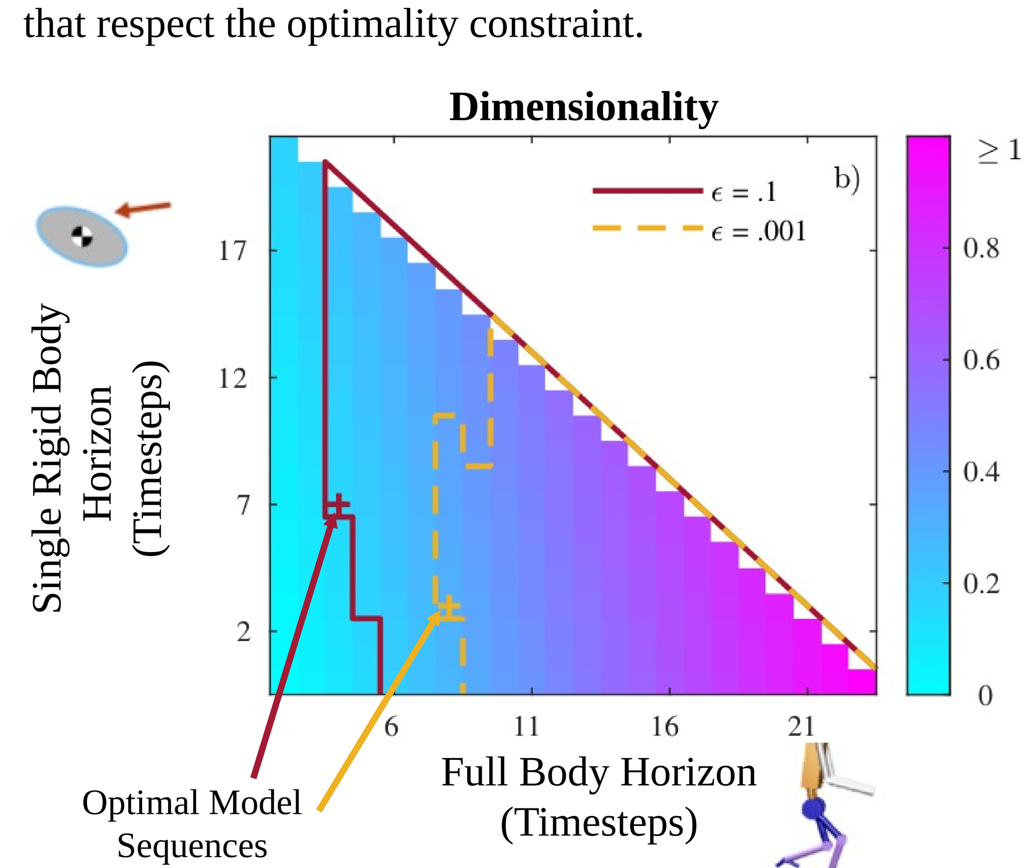
## Effect of Single Rigid Body Model Predictions

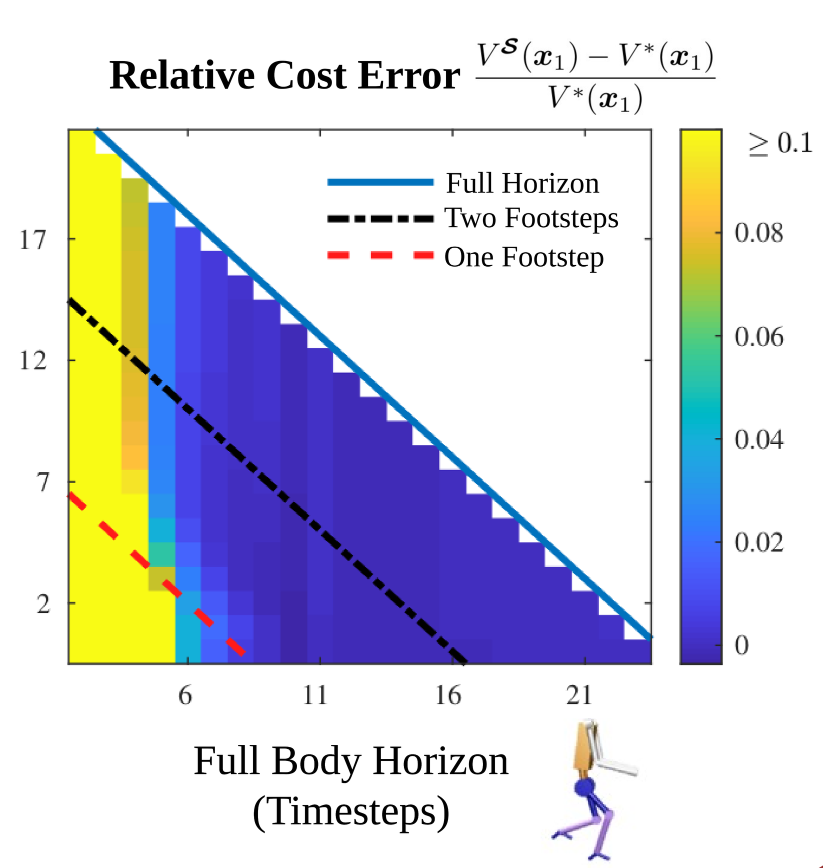
By adding the single rigid body model predictions, the closed-loop body trajectory is closer to the one obtained with full-body TO.




## Dimensionality vs Closed-Loop Cost

Multiple model sequences are near optimal. The optimal sequence is the one with the lowest dimensionality that respect the optimality constraint.




## TO vs Simulation Costs

The cost landscape varies similarly : the same local minima should be found by a search algorithm with our TO-based approximation of $V^{\mathcal{S}}(x_1)$